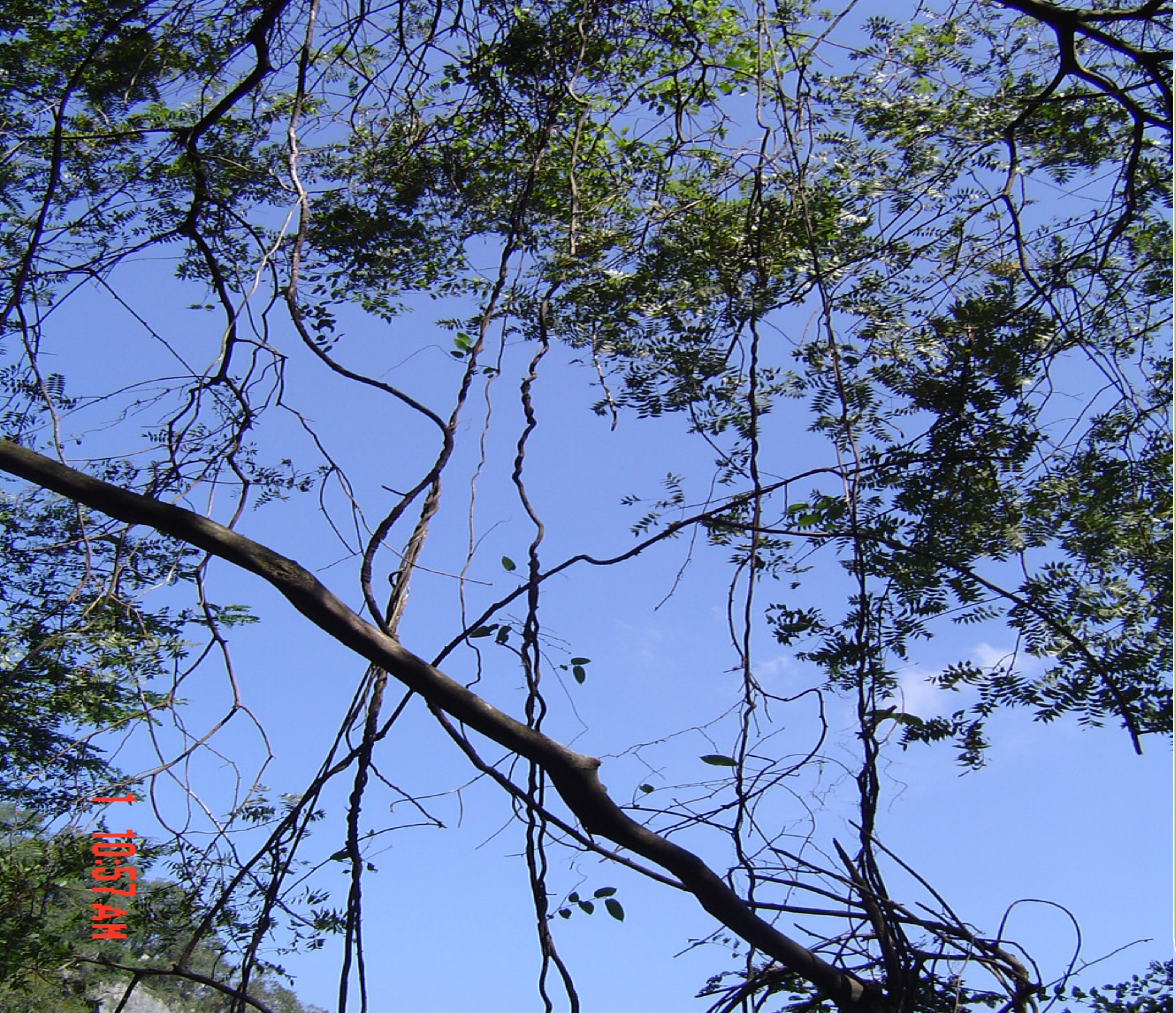
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福建师范大学数计学院

向量线性相关性的一节习题课

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- 讨论了[1]的一个习题的一系列有趣的推广, 使得学生能深刻理解线性相关性的概念, 学会提出问题的方法.
- **习题:** 设向量组$\alpha_1, \alpha_2, \alpha_3, \alpha_4$线性无关, 判断$\alpha_1+\alpha_2, \alpha_2+\alpha_3, \alpha_3+\alpha_4, \alpha_4+\alpha_1$是否也线性无关?

- **推 广1:** 设 向 量 组$\alpha_1,\alpha_2,\cdots,\alpha_n$线 性 无 关, 判 断$\alpha_1+\alpha_2,\alpha_2+\alpha_3,\cdots,\alpha_{n-1}+\alpha_n,\alpha_n+\alpha_1$是否也线性无关?

- 考虑

$$k_1(\alpha_1+\alpha_2)+k_2(\alpha_2+\alpha_3)+\cdots+k_{n-1}(\alpha_{n-1}+\alpha_n)+k_n(\alpha_n+\alpha_1)=0,$$

即

$$(k_1+k_n)\alpha_1+(k_1+k_2)\alpha_2+(k_2+k_3)\alpha_2+\cdots+(k_{n-1}+k_n)\alpha_n=0.$$

因向量组$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关, 则

$$\left\{\begin{array}{ccc} k_1+k_n & = & 0 \\ k_1+k_2 & = & 0 \\ k_2+k_3 & = & 0 \\ \cdots & \cdots & \cdots \\ k_{n-1}+k_n & = & 0 \end{array}\right. \tag{1}$$

- 齐次线性方程组(1)的系数行列式为

$$D=\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 \end{vmatrix}=1+(-1)^{n+1}$$

- 当n是奇数时, $D=2\neq 0$, 则齐次线性方程组(1)只有零解. 故$\alpha_1+\alpha_2, \alpha_2+\alpha_3, \cdots, \alpha_{n-1}+\alpha_n, \alpha_n+\alpha_1$线性无关.

- 当n是偶数时, $D=0$, 则齐次线性方程组(1)有非零解. 故$\alpha_1+\alpha_2, \alpha_2+\alpha_3, \cdots, \alpha_{n-1}+\alpha_n, \alpha_n+\alpha_1$线性相关.

- **推广2:** 若向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$使得$\alpha_1+\alpha_2, \alpha_2+\alpha_3, \cdots, \alpha_{n-1}+\alpha_n, \alpha_n+\alpha_1$ 线性无关, 判断$\alpha_1, \alpha_2, \cdots, \alpha_n$是否也线性无关?
- 考虑

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = 0,$$

$$\begin{aligned} & x_1(\alpha_1+\alpha_2) + x_2(\alpha_2+\alpha_3) + \cdots + x_{n-1}(\alpha_{n-1}+\alpha_n) + x_n(\alpha_n+\alpha_1) \\ = \; & (x_1+x_n)\alpha_1 + (x_1+x_2)\alpha_2 + (x_2+x_3)\alpha_2 + \cdots + (x_{n-1}+x_n)\alpha_n, \end{aligned}$$

令

$$\left\{ \begin{array}{ccc} x_1+x_n & = & k_1 \\ x_1+x_2 & = & k_2 \\ x_2+x_3 & = & k_3 \\ \cdots & \cdots & \cdots \\ x_{n-1}+x_n & = & k_n \end{array} \right. \qquad (2)$$

- 线性方程组(2)的系数行列式为

$$D = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 \end{vmatrix} = 1 + (-1)^{n+1}$$

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- 当n是奇数时, $D=2\neq 0$, 则线性方程组(2)有唯一解$(l_1,l_2,\cdots,l_n)^T$, 从而

$$
\begin{aligned}
& l_1(\alpha_1+\alpha_2)+l_2(\alpha_2+\alpha_3)+\cdots+l_{n-1}(\alpha_{n-1}+\alpha_n)+l_n(\alpha_n+\alpha_1) \\
= & (l_1+l_n)\alpha_1+(l_1+l_2)\alpha_2+(l_2+l_3)\alpha_2+\cdots+(l_{n-1}+l_n)\alpha_n \\
= & k_1\alpha_1+k_2\alpha_2+\cdots+k_n\alpha_n=0.
\end{aligned}
$$

  因为$\alpha_1+\alpha_2,\alpha_2+\alpha_3,\cdots,\alpha_{n-1}+\alpha_n,\alpha_n+\alpha_1$线性无关, 所以$l_1=l_2=\cdots=l_n=0$. 由线性方程组(2)得, $k_1=k_2=\cdots=k_n=0$. 故$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关.

- 当n是偶数时, 由推广1得, $\alpha_1,\alpha_2,\cdots,\alpha_n$线性相关.

- 由推广1和推广2知, 当n是奇数时, $\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关当且仅当$\alpha_1+\alpha_2,\alpha_2+\alpha_3,\cdots,\alpha_{n-1}+\alpha_n,\alpha_n+\alpha_1$线性无关.

- **推 广3:** 设 向 量 组$\alpha_1, \alpha_2, \cdots, \alpha_n$线 性 无 关, 判 断$\alpha_1 - \alpha_2, \alpha_2 - \alpha_3, \cdots, \alpha_{n-1} - \alpha_n, \alpha_n - \alpha_1$是否也线性无关?

- 考虑

$$k_1(\alpha_1 - \alpha_2) + k_2(\alpha_2 - \alpha_3) + \cdots + k_{n-1}(\alpha_{n-1} - \alpha_n) + k_n(\alpha_n - \alpha_1) = 0,$$

即

$$(k_1 - k_n)\alpha_1 + (k_2 - k_1)\alpha_2 + (k_3 - k_2)\alpha_2 + \cdots + (k_n - k_{n-1})\alpha_n = 0.$$

因向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$线性无关, 则

$$\left\{\begin{array}{rcl} k_1 - k_n & = & 0 \\ k_2 - k_1 & = & 0 \\ k_3 - k_2 & = & 0 \\ \cdots & \cdots & \cdots \\ k_n - k_{n-1} & = & 0 \end{array}\right. \tag{3}$$

- 齐次线性方程组(3)的系数行列式为

$$D=\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -1 \\ -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{vmatrix}=1+(-1)^{n+1}(-1)^n=0$$

则齐次线性方程组(3)有非零解. 故$\alpha_1-\alpha_2,\alpha_2-\alpha_3,\cdots,\alpha_{n-1}-\alpha_n,\alpha_n-\alpha_1$线性相关.

- **推广4:** 设向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$线性无关, 判断$\alpha_1+\alpha_2, \alpha_2-\alpha_3, \cdots, \alpha_{n-1}-\alpha_n, \alpha_n-\alpha_1$是否也线性无关?

- 考虑

$$k_1(\alpha_1+\alpha_2)+k_2(\alpha_2-\alpha_3)+\cdots+k_{n-1}(\alpha_{n-1}-\alpha_n)+k_n(\alpha_n-\alpha_1)=0,$$

即

$$(k_1-k_n)\alpha_1+(k_2+k_1)\alpha_2+(k_3-k_2)\alpha_2+\cdots+(k_n-k_{n-1})\alpha_n=0.$$

因向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$线性无关, 则

$$\begin{cases} k_1-k_n & = & 0 \\ k_2+k_1 & = & 0 \\ k_3-k_2 & = & 0 \\ \cdots & \cdots & \cdots \\ k_n-k_{n-1} & = & 0 \end{cases} \qquad (4)$$

• 齐次线性方程组(4)的系数行列式为

$$D = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -1 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{vmatrix} = 1 + (-1)^{n+1}(-1)^{n-1} = 2 \neq 0$$

则齐次线性方程组(4)只有零解. 故$\alpha_1 + \alpha_2, \alpha_2 - \alpha_3, \cdots, \alpha_{n-1} - \alpha_n, \alpha_n - \alpha_1$线性无关.

- **推广5:** 若向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$使得$\alpha_1+\alpha_2, \alpha_2-\alpha_3, \cdots, \alpha_{n-1}-\alpha_n, \alpha_n-\alpha_1$ 线性无关, 判断$\alpha_1, \alpha_2, \cdots, \alpha_n$是否也线性无关?

- 考虑

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = 0,$$

$$\begin{aligned} & x_1(\alpha_1+\alpha_2) + x_2(\alpha_2-\alpha_3) + \cdots + x_{n-1}(\alpha_{n-1}-\alpha_n) + x_n(\alpha_n-\alpha_1) \\ = & (x_1-x_n)\alpha_1 + (x_1+x_2)\alpha_2 + (-x_2+x_3)\alpha_2 + \cdots + (-x_{n-1}+x_n)\alpha_n, \end{aligned}$$

令

$$\left\{\begin{array}{ccc} x_1-x_n & = & k_1 \\ x_1+x_2 & = & k_2 \\ -x_2+x_3 & = & k_3 \\ \cdots & \cdots & \cdots \\ -x_{n-1}+x_n & = & k_n \end{array}\right. \tag{5}$$

- 线性方程组(5)的系数行列式为

$$D=\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & -1 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{vmatrix}=1+(-1)^{n+1}(-1)^{n-1}=2\neq 0$$

则线性方程组(5)有唯一解$(l_1,l_2,\cdots,l_n)^T$, 从而

$$\begin{aligned} & l_1(\alpha_1+\alpha_2)+l_2(\alpha_2-\alpha_3)+\cdots+l_{n-1}(\alpha_{n-1}-\alpha_n)+l_n(\alpha_n-\alpha_1) \\ = & (l_1-l_n)\alpha_1+(l_1+l_2)\alpha_2+(l_2-l_3)\alpha_2+\cdots+(l_{n-1}-l_n)\alpha_n \\ = & k_1\alpha_1+k_2\alpha_2+\cdots+k_n\alpha_n=0. \end{aligned}$$

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- 因为$\alpha_1+\alpha_2,\alpha_2-\alpha_3,\cdots,\alpha_{n-1}-\alpha_n,\alpha_n-\alpha_1$线性无关, 所以$l_1=l_2=\cdots=l_n=0$. 由线性方程组(5)得, $k_1=k_2=\cdots=k_n=0$. 故$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关.
- 由推广4和推广5知, $\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关当且仅当$\alpha_1+\alpha_2,\alpha_2-\alpha_3,\cdots,\alpha_{n-1}-\alpha_n,\alpha_n-\alpha_1$线性无关.

- **推广6:** 设向量组$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关, $k_1,k_2,\cdots,k_n$为数, 判断$\alpha_1+k_2\alpha_2,\alpha_2+k_3\alpha_3,\cdots,\alpha_{n-1}+k_n\alpha_n,\alpha_n+k_1\alpha_1$是否也线性无关?

- 考虑

$$l_1(\alpha_1+k_2\alpha_2)+l_2(\alpha_2+k_3\alpha_3)+\cdots+l_{n-1}(\alpha_{n-1}+k_n\alpha_n)+l_n(\alpha_n+k_1\alpha_1)=0,$$

即

$$(l_1+l_nk_1)\alpha_1+(l_2+l_1k_2)\alpha_2+(l_3+l_2k_3)\alpha_2+\cdots+(l_n+l_{n-1}k_n)\alpha_n=0.$$

因向量组$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关, 则

$$\left\{\begin{array}{rcl} l_1+l_nk_1 & = & 0 \\ l_2+l_1k_2 & = & 0 \\ l_3+l_2k_3 & = & 0 \\ \cdots & \cdots & \cdots \\ l_n+l_{n-1}k_n & = & 0 \end{array}\right. \qquad (6)$$

- 齐次线性方程组(6)的系数行列式为

$$D = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & k_1 \\ k_2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & k_3 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & k_{n-1} & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & k_n & 1 \end{vmatrix} = 1 + (-1)^{n+1} k_1 k_2 \cdots k_n$$

- 当$k_1 k_2 \cdots k_n = (-1)^n$时, $D = 0$, 则齐次线性方程组(6)有非零解. 故$\alpha_1 + k_2\alpha_2, \alpha_2 + k_3\alpha_3, \cdots, \alpha_{n-1} + k_n\alpha_n, \alpha_n + k_1\alpha_1$线性相关.
- 当$k_1 k_2 \cdots k_n \neq (-1)^n$时, $D \neq 0$, 则齐次线性方程组(6)只有零解. 故$\alpha_1 + k_2\alpha_2, \alpha_2 + k_3\alpha_3, \cdots, \alpha_{n-1} + k_n\alpha_n, \alpha_n + k_1\alpha_1$线性无关.
- 故$k_1 k_2 \cdots k_n = (-1)^n$当且仅当$\alpha_1 + k_2\alpha_2, \alpha_2 + k_3\alpha_3, \cdots, \alpha_{n-1} + k_n\alpha_n, \alpha_n + k_1\alpha_1$线性相关.

- **推广7:** 设$k_1, k_2, \cdots, k_n$为数, 向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$使得$\alpha_1 + k_2\alpha_2, \alpha_2 + k_3\alpha_3, \cdots, \alpha_{n-1} + k_n\alpha_n, \alpha_n + k_1\alpha_1$线性无关, 判断$\alpha_1, \alpha_2, \cdots, \alpha_n$是否也线性无关?

- 考虑

$$l_1\alpha_1 + l_2\alpha_2 + \cdots + l_n\alpha_n = 0,$$

$$\begin{aligned} & x_1(\alpha_1 + k_2\alpha_2) + \cdots + x_{n-1}(\alpha_{n-1} + k_n\alpha_n) + x_n(\alpha_n + k_1\alpha_1) \\ = \ & (x_1 + k_1x_n)\alpha_1 + (k_2x_1 + x_2)\alpha_2 + \cdots + (k_nx_{n-1} + x_n)\alpha_n, \end{aligned}$$

令

$$\left\{ \begin{array}{ccc} x_1 + k_1x_n & = & l_1 \\ k_2x_1 + x_2 & = & l_2 \\ k_3x_2 + x_3 & = & l_3 \\ \cdots & \cdots & \cdots \\ k_nx_{n-1} + x_n & = & l_n \end{array} \right. \tag{7}$$

- 线性方程组(7)的系数行列式为

$$D=\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & k_1 \\ k_2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & k_3 & 1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & k_{n-1} & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & k_n & 1 \end{vmatrix}=1+(-1)^{n+1}k_1k_2\cdots k_n$$

- 当$k_1k_2\cdots k_n \neq (-1)^n$时, $D \neq 0$, 则线性方程组(7)有唯一解$(s_1, s_2, \cdots, s_n)^T$, 从而

$$\begin{aligned} & s_1(\alpha_1 + k_2\alpha_2) + \cdots + s_{n-1}(\alpha_{n-1} + k_n\alpha_n) + s_n(\alpha_n + k_1\alpha_1) \\ = \; & (s_1 + s_nk_1)\alpha_1 + (k_2s_1 + s_2)\alpha_2 + (k_3s_2 + s_3)\alpha_2 + \cdots + (k_ns_{n-1} + s_n)\alpha_n \\ = \; & l_1\alpha_1 + l_2\alpha_2 + \cdots + l_n\alpha_n = 0. \end{aligned}$$

因为$\alpha_1 + k_2\alpha_2, \alpha_2 + k_3\alpha_3, \cdots, \alpha_{n-1} + k_n\alpha_n, \alpha_n + k_1\alpha_1$ 线性无关, 所以$s_1 = s_2 = \cdots = s_n = 0$. 由线性方程组(7)得, $l_1 = l_2 = \cdots = l_n = 0$. 故$\alpha_1, \alpha_2, \cdots, \alpha_n$线性无关.

- 当$k_1k_2\cdots k_n = (-1)^n$时, 由推广6得, $\alpha_1, \alpha_2, \cdots, \alpha_n$线性相关.
- 由推广6和推广7得, 当$k_1k_2\cdots k_n \neq (-1)^n$时, $\alpha_1, \alpha_2, \cdots, \alpha_n$线性无关当且仅当$\alpha_1 + k_2\alpha_2, \alpha_2 + k_3\alpha_3, \cdots, \alpha_{n-1} + k_n\alpha_n, \alpha_n + k_1\alpha_1$ 线性无关.

- **推广8:** 设向量组$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关, 且

$$\begin{cases} \beta_1 &= a_{11}\alpha_1 + a_{12}\alpha_2 + \cdots + a_{1n}\alpha_n \\ \beta_2 &= a_{21}\alpha_1 + a_{22}\alpha_2 + \cdots + a_{2n}\alpha_n \\ & \cdots\cdots\cdots \\ \beta_n &= a_{n1}\alpha_1 + a_{n2}\alpha_2 + \cdots + a_{nn}\alpha_n \end{cases},$$

判断$\beta_1,\beta_2,\cdots,\beta_n$是否也线性无关?

- 考虑

$$k_1\beta_1 + k_2\beta_2 + \cdots + k_n\beta_n = 0,$$

即

$$(k_1a_{11} + k_2a_{21} + \cdots + k_na_{n1})\alpha_1 + (k_1a_{12} + k_2a_{22} + \cdots + k_na_{n2})\alpha_2 + \cdots + (k_na_{1n} + k_2a_{2n} + \cdots + k_na_{nn})\alpha_n = 0.$$

由于$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关, 则

$$\begin{cases} k_1a_{11} + k_2a_{21} + \cdots + k_na_{n1} &= 0 \\ k_1a_{12} + k_2a_{22} + \cdots + k_na_{n2} &= 0 \\ \cdots\cdots\cdots & \\ k_na_{1n} + k_2a_{2n} + \cdots + k_na_{nn} &= 0 \end{cases}, \tag{8}$$

- 若

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \neq 0,$$

则齐次线性方程组(8)只有零解, 故$\beta_1, \beta_2, \cdots, \beta_n$线性无关.

- 若

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0,$$

则齐次线性方程组(8)有非零解, 故$\beta_1, \beta_2, \cdots, \beta_n$线性相关.

- **推广9:** 对于向量组$\alpha_1, \alpha_2, \cdots, \alpha_n$, 令

$$\begin{cases} \beta_1 &= a_{11}\alpha_1 + a_{12}\alpha_2 + \cdots + a_{1n}\alpha_n \\ \beta_2 &= a_{21}\alpha_1 + a_{22}\alpha_2 + \cdots + a_{2n}\alpha_n \\ & \cdots\cdots\cdots \\ \beta_n &= a_{n1}\alpha_1 + a_{n2}\alpha_2 + \cdots + a_{nn}\alpha_n \end{cases},$$

若$\beta_1, \beta_2, \cdots, \beta_n$线性无关, 判断$\alpha_1, \alpha_2, \cdots, \alpha_n$是否也线性无关?

- 考虑

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = 0,$$

和

$$\begin{aligned} & x_1\beta_1 + x_2\beta_2 + \cdots + x_n\beta_n \\ = & (x_1a_{11} + x_2a_{21} + \cdots + x_na_{n1})\alpha_1 + (x_1a_{12} + x_2a_{22} + \cdots + x_na_{n2})\alpha_2 \\ & + \cdots + (x_na_{1n} + x_2a_{2n} + \cdots + x_na_{nn})\alpha_n. \end{aligned}$$

令

$$\left\{\begin{array}{lcl} x_1a_{11} + x_2a_{21} + \cdots + x_na_{n1} & = & k_1 \\ x_1a_{12} + x_2a_{22} + \cdots + x_na_{n2} & = & k_2 \\ \cdots\cdots\cdots & & \\ x_na_{1n} + x_2a_{2n} + \cdots + x_na_{nn} & = & k_n \end{array}\right., \qquad (9)$$

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$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \neq 0,$$

则线性方程组(9)有唯一解$(l_1, l_2, \cdots, l_n)^T$, 从而

$$\begin{aligned} & l_1\beta_1 + l_2\beta_2 + \cdots + l_n\beta_n \\ = & (l_1a_{11} + l_2a_{21} + \cdots + l_na_{n1})\alpha_1 + (l_1a_{12} + l_2a_{22} + \cdots + l_na_{n2})\alpha_2 + \cdots \\ & +(l_na_{1n} + l_2a_{2n} + \cdots + l_na_{nn})\alpha_n \\ = & k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = 0 \end{aligned}$$

因$\beta_1, \beta_2, \cdots, \beta_n$线性无关, 则$l_1 = l_2 = \cdots = l_n = 0$, 由线性方程组(9)知, $k_1 = k_2 = \cdots = k_n = 0$, 故$\beta_1, \beta_2, \cdots, \beta_n$线性无关.

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$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = 0,$$

则$\alpha_1, \alpha_2, \cdots, \alpha_n$线性相关.

References

[1] 丘维声. 高等代数. 高等教育出版社. 2002.

谢谢!